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Comments on "A New Concept for Broadbanding the Ferrite Substrate Circulator Based on Experimental Modal Analysis"

STEVEN MARCH

Abstract—**In the above paper,[1] the authors concluded, from a modal analysis of microstrip ferrite circulators, that the combination of two adjacent modes should be used for broad-banding circulators, which do not require quarter-wavelength matching transformers. Upon careful examination of their results, it can be shown that Miura and Hashimoto actually produced conventional below-resonance circulators—transformers included.**

Recently, Miura and Hashimoto proposed that broad-band microstrip circulators on ferrite substrates, without conventional matching transformers, can be realized by a combination of the $n = 1$ mode, as proposed by Bosma [2] and Fay and Comstock [1], and a separate, nonresonant mode. This second mode would be different from, but compatible with, the former one. Instead of magnetizing just the area enclosed by the shield diameter d a region of substantially larger diameter D was magnetized (Fig. 1). The optimum results were obtained from equi-isolation curves; the optimum ratio was $D/d = 1.90$. The optimum w/h was found to be 1.0. These results were obtained for a YIG substrate having a saturation magnetization $4\pi M_s$ of 1750 G. The experimental circulator had a center frequency of 6.3 GHz. Since Miura and Hashimoto did not present values of the relative dielectric constant of their YIG substrate nor the bias magnetization, a relative permittivity ϵ_r of 16.0 and a magnetization of 1250 G (approximately equal to the remanence magnetization) will be assumed.

CIRCULATORS AND RESONATORS

In the design of conventional Y-junction circulators, investigators have assumed that the z-component of the electric field traverses the periphery of the ferrite disk in only one period, i.e., only the $n = 1$ mode is present. For a below-resonance circulator operating in this mode, Fay and Comstock have found the diameter of the ferrite puck D_f to be related to the free-space wavelength λ_0 by:

$$D_f = \frac{0.586\lambda_0}{(\epsilon_{\text{eff}}\mu_{\text{eff}})^{1/2}} \tag{1}$$

where ϵ_{eff} and μ_{eff} are the effective permittivity and effective permeability of the ferrite, respectively.

Watkins [3], in studying the modal patterns of disk sections in microstrip, concluded that the $n = 1$ mode is dominant because, for a given frequency, it requires a minimum diameter. This resonator mode is also characterized by (1). Thus there exists a duality between circulators and circular disk resonators in microstrip.

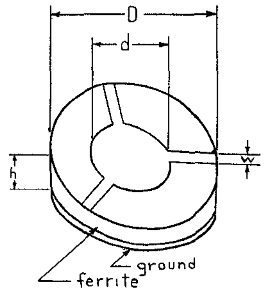

Fig. 1. Ferrite circulator geometry.

In his study of circular resonators for microwave integrated circuits, Schwarzmann [4] reported the quality of the principal resonance to be poor, indicating a low value of Q. He found the following analytic expression for the unloaded Q of the disk resonator,

$$Q_u = 120(f_{\text{GHz}})^{1/2}. \tag{2}$$

Schwarzmann also reported the resonant frequency of the principal mode to be slightly lower than predicted by (1). This is due to fringe effects, not accounted for in the theoretical analysis, which decrease the resonant frequency. The effect of radiation is similar to fringing, but of lesser magnitude. Since the principle resonance is equivalent to a TM_{110} mode, the presence of a conducting ground perturbs the magnetic field and tends to increase the resonant frequency [5]. As reasonable estimates the combined effects of fringing and radiation would warrant a reduction in diameter by 8 percent, offset 3 percent by the magnetic field perturbations. Thus (1) should be used to define an effective ferrite or resonator diameter. A reasonably accurate estimate of the physical diameter would be

$$D_f' = \frac{0.557\lambda_0}{(\epsilon_{\text{eff}}\mu_{\text{eff}})^{1/2}} = 0.557\,\lambda_g. \tag{3}$$

How (3) relates to the work of Miura and Hashimoto will be made clear in the following.

CIRCULATOR DESIGN

If we set D_f' equal to d of Fig. 1, and if we assume conventional circulator design which includes quarter-wave matching transformers, then it would be reasonable to conclude

$$D = d + 2\left(\frac{\lambda_g}{4}\right). \tag{4}$$

Then, from (3) and (4), it follows that D is 1.057 λ_g and the ratio D/d equals 1.898, in excellent agreement with the optimum results of Miura and Hashimoto. Thus the ratio D/d due to Miura and Hashimoto is nearly identical to that of conventional circulator design, assuming the presence of the $n = 1$ mode only.

As would be expected, the dimensions of Miura and Hashimoto for a circulator at 6.3 GHz also conform with established theory. In a microstrip media, the effective permittivity must be used in lieu of the relative dielectric constant. To determine the effective dielectric constant, (5) due to Schneider [6] yields highly accurate results at zero frequency

$$\epsilon_{\text{eff}_0} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\,\frac{1}{(1 + 10h/w)^{1/2}}. \tag{5}$$

For the disk structure at 6.3 GHz, Miura and Hashimoto used a shield diameter of 8 mm and a substrate thickness h of 1.5 mm. In calculating the zero-frequency effective dielectric constant of the disk structure, a w/h of 2.67, $(1/2)d/h$, was assumed; for the connecting lines, a w/h of 1.0 was used. Equation (5) gives effective permittivities of 10.76 and 11.94 for the connecting lines and disk, respectively.

Dispersion in microstrip increases the effective dielectric constant with increasing frequency. Schneider [7] has also studied the frequency dependence of the effective permittivity. In a modified form,

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The author is with Andersen Laboratories, Inc., Bloomfield, Conn. 06002.
[1] T. Miura and T. Hashimoto, in *1971 IEEE G-MTT Symp. Dig.*, pp. 80–81.

Schneider's equation for microstrip dispersion is

$$\epsilon_{\text{eff}} = \left[\frac{(f/f_c)^2 + 1}{(\epsilon_r)^{-1/2}(f/f_c)^2 + (\epsilon_{\text{eff0}})^{-1/2}}\right]^2 \qquad (6)$$

where f_c is the divergence frequency, given by

$$f_c = \frac{c_0}{4h(\epsilon_r - 1)^{1/2}} \qquad (7)$$

and c_0 is the velocity of light in air. For the two conditions of w/h mentioned in the preceding paragraph, $w/h = 1.0$ and $w/h = 2.67$, the effective permittivities at 6.3 GHz are 11.56 and 12.59, respectively.

In a recent paper, Schlöman [8] has shown that the permeability tensor element as a function of frequency for the demagnetized state can be approximated by

$$\mu_{\text{dem}} = \frac{1}{3} + \frac{2}{3}\left\{1 - \left[\frac{\gamma(4\pi M_s)}{f_{\text{MHz}}}\right]^2\right\}^{1/2} \qquad (8)$$

where γ is the gyromagnetic ratio equal to 2.8 MHz/Oe. Using (8), a value of 0.750 for the permeability of the demagnetized YIG at 6.3 GHz is obtained. Green and his co-workers [9] have proposed the following equation for the relative permeability of partially magnetized substrates:

$$\mu_{\text{mag}} = \mu_{\text{dem}} + \left(\frac{4\pi M}{4\pi M_s}\right)^{3/2}(1 - \mu_{\text{dem}}). \qquad (9)$$

In (9), $4\pi M$ is the magnetization through the ferrite and becomes the remanent value $4\pi M_r$ when latched into remanence. In our example μ_{mag} equals 0.900.

Using a duality relationship for dielectric and magnetic substrates, the effective permeability of ferrite substrates for microstrip transmission has been treated by Massé and Pucel [10]. They found for $w/h \leq 2$

$$\mu_{\text{eff}} = \frac{2\mu_{\text{mag}}}{1 + \mu_{\text{mag}}}\left(\frac{A - B}{A}\right)^2 \qquad (10)$$

and

$$A = \ln\frac{8h}{w} + \frac{1}{32}\left(\frac{w}{h}\right)^2$$

$$B = \frac{1}{2}\left(\frac{1 - \mu_{\text{mag}}}{1 + \mu_{\text{mag}}}\right)\left[\ln\frac{\pi}{2} + \mu_{\text{mag}}\ln\frac{4}{\pi}\right].$$

For the disk section in question $\mu_{\text{eff}} = 0.890$, while $\mu_{\text{eff}} = 0.923$ for the $w/h = 1.0$ condition.

Using the results of (6) and (10) in (3), a disk section at 6.3 GHz would have a diameter d of 7.93 mm, in excellent agreement with the results of Miura and Hashimoto considering the accuracy of the equations and variables used in the calculations.

For the three series transmission lines used for the three ports, the characteristic impedance can be computed from Schneider's

$$Z_0 = 60\left(\frac{\mu_{\text{eff}}}{\epsilon_{\text{eff}}}\right)^{1/2}\ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \qquad (11)$$

for $w/h \leq 1$. For the case in study, Z_0 works out to 35.8 Ω, not unrealistic for circulator matching transformer impedances. The electrical length of these segments, using D of 15 mm, is 0.240 λ_g, again in good agreement with conventional circulator design.

From all of the above, we can conclude that Miura and Hashimoto have, in reality, not introduced a new concept, but verified an existing one.

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Authors' Reply[2]

TARO MIURA AND TADASHI HASHIMOTO

Abstract—**We have demonstrated that there emerges a new mode consecutive to the $n = 1$ mode, which is effective in realizing a compact broad-band circulator. Recently, March has presented a comment against this point while ascribing the broad-banding to the combined effect of a junction circulator and matching networks. Reexamination, however, has proved the cited mode actually exists.**

In general, broad-band ferrite substrate circulators of the below-resonance operation comprise the Y-junction contributing to the circulation and quarter-wavelength transformers disposed on ferrite substrates. In the above paper,[1] we have demonstrated that the broad-banding of this class of circulator can also be realized by an appropriate choice of the junction diameter to ferrite disk diameter ratio. After reasoning on the basis of the experimental 20-dB equi-isolation curve in conjunction with probed field patterns, we have attributed a sizeable additional broad-banding to a new mode which may well be designated as the wall-affected mode (tentatively abbreviated as WA mode) and which is excited following the $n = 1$ mode, as proposed by Fay and Comstock [1], as the frequency is lowered. Definitely present as it is, this WA mode is not amenable to an analysis based on an appropriate modeling, so that this mode has been identified to be distinct from the conventional mode on the basis of field patterns as experimentally probed.

Against our point allowing for the consecutive excitation of this new mode, March has presented an argument to the effect that a perfect interpretation may be achieved on the performance of this circulator from conventional circulator theory. In response to his argument, we have worked out our experimental results again, while extending the range where experimental measurements are carried out and performing the field probing at the same time.

The variables affecting the circulator characteristics are the diameter of ferrite disk D, the diameter of junction conductor d, the thickness of substrate h, and the stripline width w, shown in Fig. 1. Typical 20-dB equi-isolation curves on the frequency-magnetic field strength plane are shown in Fig. 2. As shown in the figure, the relative 20-dB isolation bandwidth is maximized to 50 percent for D/d 1.9 and a dc field of 1250 Oe. In addition, surface electric field measurements show that the $n = 1$ mode is dominant from the central frequency to the higher band edge while the WA mode is excited near the lower band edge. Summarizing these: 1) the WA mode is excited in the magnetized region so that the operational range of a circulator extends downward; 2) the ratio (D/d) should be larger than 1.9 to excite the WA mode which is consecutive to the $n = 1$ mode; and 3) the broad banding of this experimental circulator is realized by the frequency sharing of the $n = 1$ mode and the WA mode.

Additional Experimental Observation

In order to reinforce our stand, a series of surface electric field measurements were again performed for the value of $(D/d) = 2.0$. As evidenced in Fig. 2 this value causes the WA mode to just separate from the $n = 1$ mode. The WA mode and the $n = 1$ mode

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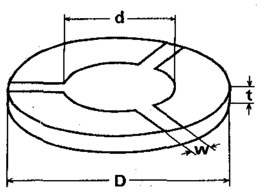

Fig. 1. Ferrite circulator geometry.

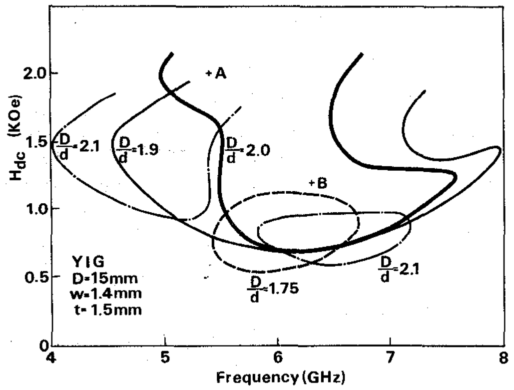

Fig. 2. 20-dB equi-isolation curves on magnetic field strength-frequency plane.

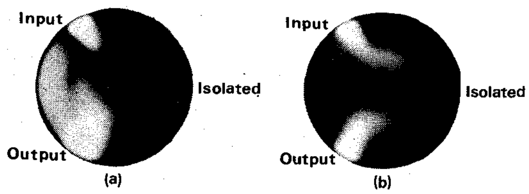

Fig. 3. Probed electric field patterns. (a) H_{dc} = 2000 Oe. Frequency = 5.4 GHz, WA mode. (b) H_{dc} = 1200 Oe. Frequency = 6.3 GHz, n = 1 mode.

prevail in the upper and lower halves, respectively, of the domain bounded by the curve labeled as $(D/d) = 2.0$. The electric field distributions are measured for two different conditions including $H_{dc} = 2000$ Oe, $f = 5.4$ GHz, and $H_{dc} = 1200$ Oe, $f = 6.3$ GHz, indicated as A and B in Fig. 2, respectively. Two distinctly different field patterns are obtained as shown in Fig. 3. The field configuration for the point B as reproduced in Fig. 3 shows a close fit to the prediction on the basis of a TM_{110} mode. By contrast the field pattern for the point A as referred to above shows no symmetry with respect to an axis passing through the isolated port, concentrating on the region ranging from the input to the output ports. These findings permit the conclusion to be reached again that the realized broadbanding may be ascribed to the frequency sharing by two different but consecutive modes.

As pointed out by March, the magnetized microstrip should be considered as an impedance-matching element. But the characteristic impedance and the electric length of the microstrip are too high and too short, respectively, to serve as a quarter-wavelength transformer. The reason is stated as follows. Let us restrict the dimensions of the circulator to $D = 15$ mm, $d = 8$ mm, $w = 1.5$ mm, and $t = 1.5$ mm for a relative bandwidth of 50 percent, under the applied magnetic field of 1250 Oe. Yttrium iron garnet, which has a saturation magnetization of 1750 G, a dielectric constant of 15, and a remanent magnetization assumed to be 1250 G, is utilized. After March the effective dielectric constant (ϵ_{eff}) is calculated to be 10.11 for w/h of 1.0 and frequency of 6.3 GHz. Using March's (8) a value of 0.752 for μ_{dem} is obtained.

Because the applied magnetic field is perpendicular to the ferrite disk, the demagnetizing field should be taken into account. For the thin ferrite remanent, magnetization will be reached when an external dc magnetic field (H_{dc}) is applied, which is numerically equal to the saturation magnetization ($4\pi M_s$). Therefore, for $H_{dc} \leq 4\pi M_s$, the magnetization may be approximately expressed as

$$4\pi M = 4\pi M_r \frac{H_{dc}}{4\pi M_s}. \quad (1)$$

For our case this gives a magnetization of 900 G and $\mu_{mag} = 0.843$. For the microstrip section $\mu = \mu_{mag}$, since the demagnetizing field for the RF field is negligible. This gives an effective permeability of 0.891 from (10) above. The wavelength in this section is 1.53 cm at 6.3 GHz, which results in a microstrip section becoming 0.228 λ_g long.

Now, μ_{mag} increases with frequency causing the electric length of this section to change at a rate much greater than that for the dielectric substrate microstrip. According to an analysis of broad-band circulators presented by Anderson [5], a necessary impedance for a quarter-wavelength transformer is obtained as

$$\frac{Z_0}{Z_t} = \rho_m \left(1 + \frac{4}{\pi} \frac{(1-\rho_m)^{1/2}}{\Delta f/f}\right)^{1/2} \quad (2)$$

where ρ_m is the maximum allowable standing-wave ratio at the input port, $\Delta f/f$ is the attainable bandwidth, and Z_0 is the characteristic impedance of the external circuit (50 Ω in this example). Hence the characteristic impedance of the transformer is calculated to be 25.8 Ω when the relative bandwidth is assumed to be 50 percent, in disagreement with the characteristic impedance of 35.8 Ω found from Schneider's equation in [6] by March. Thus the description of the circulator based exclusively on matching theory, as offered by March, is not comprehensive enough to cover all the behavior of circulators of interest.

Summarizing the entire description, apart from our comment against the remarks on the matching section and so forth by March, we would like to claim that we are justified in saying that we have noted a new circulator mode playing a substantial role in extending the operational bandwidth, and that we have set forth a new design concept for circulators.

Acknowledgment

The authors wish to thank Dr. N. Ogasawara of the Tokyo Metropolitan University for his stimulating discussions.

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Reply[3] *by S. March*

STEVEN MARCH

In their rebuttal, as in their original manuscript, Miura and Hashimoto claim that two distinct consecutive modes must be present in order to be able to produce a broad-band circulator in a microstrip configuration. These are the $n = 1$ mode as proposed by Fay and Comstock [2] and a new mode, which the authors have called a "wall-affected" mode.

It is felt, however, that established theory requires, as a condition for proper Y-junction circulation, the existence of two distinct modes propagating simultaneously. Bosma's theory for Y-junction circulators states that the application of a magnetic biasing field to

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the circular disk structure establishes two counter-rotating modes, the $n = \pm 1$ modes, which are necessary for Y-circulation. When these modes are established, the operating center frequency must lie between the modal resonant frequencies [1].

In their rebuttal, Miura and Hashimoto disagree with this author on the type of mode resonance established in the circular disk structure. This is not difficult to understand, since the authors of various articles dealing with circular resonators in microstrip and Y-junction circulators do not seem to agree on the nature of the lowest order, or dominant, mode resonance. In the case of stripline and microstrip circulators, Bosma [3] has stated that the most elementary mode is the TM_{110} mode. Fay and Comstock recognized that the lowest frequency resonance in the operation of a Y-junction circulator is in the dipolar mode. The dipolar mode is usually considered the TE_{01} mode. Schwarzmann [4] has explained that the fundamental mode of a resonant disk structure in microstrip is a pseudo-TE_{010} mode with a near-perfect H-barrier perpendicular to the edge of the disk.

Watkins [5], who also studied circular disk resonators in microstrip, has shown that the $n = 1$ mode is dominant. In arriving at his results, Watkins employed TM-mode analysis. It is interesting to note, however, that the diameters required to establish resonance in the $n = 0$ through $n = 3$ modes agree exactly with the necessary diameters for the corresponding TE_{n1} mode in circular waveguide. Indeed, the diameter required to establish Schwarzmann's pseudo-TE_{010} mode is identical to that required for Watkins's $n = 1$ mode, which in turn is equal to the required diameter for Fay and Comstock's dipolar mode and Bosma's TM_{110} mode.

Furthermore, it is logical to assume TM-mode propagation in either the circular disk resonator or the Y-junction circulator, since the axial component of the magnetic field, H_z, is equal to zero in both instances. However, the electric field contours as measured by Miura and Hashimoto and as determined by Bosma (see [1, fig. 6]) more closely resemble a degeneracy of the TE_{110} mode than the TM_{110} mode.

Establishment of the dominant $n = 1$ mode resonance in a circular disk structure requires a diameter of $0.586\ \lambda_g$. For a below-resonance circulator operating in the $n = 1$ mode, Fay and Comstock arrived at the same diameter. This is the same diameter which is required to establish TE_{11} mode resonance in a circular waveguide. In a microstrip medium, it should be considered an effective diameter, since the effects of fringing, radiation, and ground perturbations must be included. Experimentally, Schwarzmann found the resonant frequency of the disk structure to be slightly lower than predicted by elementary theory for the reasons mentioned. This effect has also been observed in the performance of circulators constructed by Bosma [3]. When the effects of radiation, ground perturbations, and fringing are included, the physical diameter of the circular structure should be $0.557\ \lambda_g$. Assuming that D is one-half wavelength larger than d, the ratio D/d is found to be 1.898, which is nearly identical to the optimum ratio of 1.90 found by Miura and Hashimoto.

Referring again to their rebuttal, Miura and Hashimoto calculated the electrical lengths of the microstrip connecting sections using an effective permittivity of 10.11 and an effective permeability of 0.891 at 6.3 GHz. This value of ϵ_{eff} is applicable for a relative dielectric constant of 15 at zero frequency, and does not include the effects of dispersion on the effective dielectric constant. Once included, ϵ_{eff} becomes 10.8. At 6.3 GHz, a quarter-wavelength in the microstrip media is 0.367 cm. Stated in a different fashion, these microstrip sections represent 0.238 guide wavelengths, and not 0.228 wavelengths as rebutted by Miura and Hashimoto. The question then arises as to where is the electrical location of the reference plane from which these electrical lengths are measured. It is not unreasonable to assume that the electrical reference plane exists slightly within the boundary of the disk, which represents the lower of the two impedances. This correction need be only 6.7 mil in order that the electrical lengths of the connecting sections be a quarter-wavelength.

Since it was pointed out that the dielectric constant of YIG should be 15 and not 16, it was necessary to recalculate the diameter required for resonance at 6.3 GHz. The calculations yielded the following data: $\epsilon_{\text{eff}_0} = 11.21$; $\epsilon_{\text{eff}} = 11.785$; $\mu_{\text{dem}} = 0.750$; $\mu_{\text{mag}} = 0.842$; $\mu_{\text{eff}} = 0.955$; and $\lambda_g = 1.419$ cm. The calculation of μ_{mag} included the correction for demagnetizing fields as suggested by Miura and Hashimoto. In the calculation of μ_{eff}, however, equations valid for the condition $w/h \geq 2$ were used. These equations, due to Massé and Pucel [6], are as follows:

$$\mu_{\text{eff}} = \mu_{\text{mag}} \left[\frac{C}{C - D} \right]^2 \tag{1}$$

$$C = \frac{w}{2h} + \frac{1}{\pi} \left[\ln 2\pi\epsilon \left(\frac{w}{2h} + 0.94 \right) \right] \tag{2}$$

$$D = \frac{1 - \mu_{\text{mag}}}{2} \left\{ \ln \left[\frac{\pi\epsilon}{2} \left(\frac{w}{2h} + 0.94 \right) \right] - \mu_{\text{mag}} \ln \left(\frac{\pi\epsilon^2}{16} \right) \right\} \tag{3}$$

where ϵ is defined by $\ln \epsilon = 1$. The physical diameter, $0.557\lambda_g$, is thus calculated to be 7.90 mm, within 2 percent of the physical diameter due to Miura and Hashimoto.

In determining the characteristic impedance required for a quarter-wave transformer to match the circulator junction to 50 Ω, Miura and Hashimoto referred to a recent article by Anderson [7]. Unfortunately, (5) due to Miura and Hashimoto is incorrect, probably due to a transcription error. This is a "compound" error, however, since the referenced equation is also erroneous. Anderson, in transposing the referenced equation from his Appendix, where it is derived, omitted a squared factor. The correct equation should read:

$$\frac{Z_0}{Z_T} = \rho_m \left[1 + \frac{2(\rho_m - 1)}{b^2} \right]^{1/2} \tag{4}$$

where ρ_m is the maximum VSWR over the bandwidth Δf. The normalized factor b is given by

$$b = \frac{\pi}{4} \left(\frac{\Delta f}{f_0} \right). \tag{5}$$

It should also be mentioned that (4), above, is a simplification of a more exact result derived by Anderson. The required characteristic impedance for a quarter-wave impedance-matching transformer when calculated using (4) is approximately 10 percent lower than when derived using Anderson's more exact equation. Assuming a relative bandwidth of 50 percent and a maximum VSWR of 1.21 (isolation of 20 dB), Z_T, as calculated from (4) is 19.5 Ω. The more exact equation yields a Z_T of 21.4 Ω. Both of these results are lower than the 25.8 Ω calculated by Miura and Hashimoto.

At 6.3 GHz, a 26-Ω microstrip line on a demagnetized substrate having a relative permittivity of 15 requires a w/h ratio of 2.2, far removed from the w/h ratio of 1.0 used by Miura and Hashimoto to achieve a broad-band match. For a demagnetized YIG substrate, a w/h of 1.0 corresponds to an impedance level of 36.4 Ω at 6.3 GHz.

In another article on the broad-band impedance-matching of a Y-junction circulator, Helsjahn [8] presents a different set of equations for the calculation of the characteristic impedance of the matching transformer. Using Helsjahn's equations, this impedance level is calculated to be 35.4 Ω, in close agreement with a w/h of 1.0.

Thus it has again been shown that Miura and Hashimoto have actually produced conventional below-resonance circulators based upon Fay and Comstock's $n = 1$ mode. Their microstrip connecting sections are, in reality, quarter-wave impedance-matching transformers.

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